

DISTRIBUTED COMPUTING ON THE ENEA GRID: TWO APPLICATIONS FROM SCIENCE AND ENGINEERING

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We study two mathematical models concerning two applications taken from science and engineering. The mathematical models have been solved using highly parallelizable numerical methods. These numerical methods have been implemented on the ENEA Grid using distributed computing through the JOB ARRAY instruction of LSF. The first application (acoustic obstacle scattering) uses also parallel computing through the message passing library MPI. The science application concerns acoustic obstacle scattering, the engineering application concerns cascading blackouts in a power transmission network.

1. Acoustic obstacle scattering

We consider the following problem [1], [2], [3]:

Given an acoustic incoming plane wave, not necessarily time harmonic propagating, with wave propagation velocity $c > 0$ in a homogeneous isotropic medium at equilibrium, with no source terms present and an obstacle Ω compute the acoustic field scattered by the obstacle when hit by the incoming wave.

The obstacle is assumed to be a bounded simply connected open set with locally Lipschitz boundary $\partial\Omega$ contained in the three dimensional Euclidean space and to be characterized by a constant acoustic boundary impedance $\chi \geq 0$. The incoming wave $u^i(\underline{x}, t)$, $(\underline{x}, t) \in \mathbf{R}^3 \times \mathbf{R}$ is assumed to satisfy the wave equation (1) in $\mathbf{R}^3 \times \mathbf{R}$. The obstacle (Ω, χ) when hit by the incoming wave $u^i(\underline{x}, t)$, $(\underline{x}, t) \in \mathbf{R}^3 \times \mathbf{R}$ generates a scattered wave $u^s(\underline{x}, t)$, $(\underline{x}, t) \in (\mathbf{R}^3 \setminus \overline{\Omega}) \times \mathbf{R}$ solution of the following exterior problem for the wave equation:

$$(1) \quad \Delta u^s(\underline{x}, t) - \frac{1}{c^2} \frac{\partial^2 u^s}{\partial t^2}(\underline{x}, t) = 0, \quad (\underline{x}, t) \in (\mathbf{R}^3 \setminus \overline{\Omega}) \times \mathbf{R},$$

with the boundary condition:

$$(2) \quad -\frac{\partial u^s}{\partial t}(\underline{x}, t) + c\chi \frac{\partial u^s}{\partial \underline{n}(\underline{x})}(\underline{x}, t) = g(\underline{x}, t), \quad (\underline{x}, t) \in \partial\Omega \times \mathbf{R},$$

where $g(\underline{x}, t)$ is given by:

$$(3) \quad g(\underline{x}, t) = \frac{\partial u^i}{\partial t}(\underline{x}, t) - c\chi \frac{\partial u^i}{\partial \underline{n}(\underline{x})}(\underline{x}, t), \quad (\underline{x}, t) \in \partial\Omega \times \mathbf{R},$$

the boundary condition at infinity:

$$(4) \quad u^s(\underline{x}, t) = O\left(\frac{1}{r}\right), \quad r \rightarrow +\infty, \quad t \in \mathbf{R},$$

and the radiation condition:

$$(5) \quad \frac{\partial u^s}{\partial r}(\underline{x}, t) + \frac{1}{c} \frac{\partial u^s}{\partial t}(\underline{x}, t) = o\left(\frac{1}{r}\right), \quad r \rightarrow +\infty, \quad t \in \mathbf{R},$$

where $\underline{x} = (x_1, x_2, x_3)^T \in \mathbf{R}^3$, the superscript T means transposed, $r = \|\underline{x}\| = (x_1^2 + x_2^2 + x_3^2)^{1/2}$, $\underline{x} \in \mathbf{R}^3$, $\Delta = \sum_{i=1}^3 \frac{\partial^2}{\partial x_i^2}$ is the Laplace operator, $c > 0$ is the wave propagation velocity and $O(\cdot)$ and $o(\cdot)$ are the Landau symbols. An obstacle $(\Omega; \chi)$ that when hit by the incoming wave u^i generates a scattered wave u^s solution of (1)-(5) sometime is called passive obstacle.

The algorithm proposed to solve (1)-(5) is a generalization of the “operator expansion method” developed in [1] and it reformulates the exterior problem for the wave equation using a perturbation approach. More specifically the algorithm proposed assumes that the incoming wave and the scattered wave can be represented as a superposition of time harmonic plane waves and reduces the solution of the time dependent problem for the wave equation to the solution of a set of time harmonic scattering problems, that is to the solution of a set of exterior boundary value problems for the Helmholtz equation, one problem for each frequency in the spectrum of the incoming wave.

Next we assume that the space dependent part of the solution of each time harmonic problem for the Helmholtz equation can be represented as a single layer potential with a density (that must be determined) defined on the boundary of a suitable obstacle Ω_c contained in Ω . The algorithm reduces the solution of each exterior boundary value problem for the Helmholtz equation to the solution of a set of integral equations defined on the boundary of Ω_c and of a “reference” obstacle Ω_r .

The numerical difficulty of these exterior problems can be measured using the parameter R_T defined as the ratio between the characteristic dimension of the obstacle and the wavelength of the time harmonic component of the acoustic incoming wave considered. The parameter R_T is an a-dimensional quantity and large values of R_T correspond to high frequency scattering, that is to numerically difficult problems.

In the numerical experiments presented in [1], [2], [3], the ratios R_T considered range in the interval [3,100]. This implies the use of high dimensional vector spaces to approximate satisfactorily the corresponding systems of integral equations. That is each system of integral equations in the

sequence mentioned above after being discretized to be solved numerically becomes a large system of linear equations.

Represented on a generic base the matrices that approximate the integral operators after the discretization will be dense matrices. The solution of large dense linear systems will become computationally (computing time and memory requirements) unaffordable even for moderate values of R_T .

A new way of using the wavelet transform and new bases of wavelets have been introduced and a version of the operator expansion method that constructs directly element by element the coefficient matrix of the sparse linear systems that approximate in the wavelet basis considered the systems of integral equations has been developed.

The resulting numerical method has been implemented on the ENEA Grid [4] and is able to deal with realistic acoustic scattering problems solving large (sparse) linear systems (up to approximately $5 \cdot 10^5$ (real)) unknowns and equations.

These results are obtained running simultaneously clusters of 500 jobs on the ENEA Grid. The computation of the elements of the coefficient matrices of the linear systems considered and several other aspects of the numerical algorithm proposed can be carried out independently one from the other allowing the development of an efficient solver for exterior boundary value problems for the Helmholtz equation, particularly suited to be executed on a computing grid using distributed computing. The solution of the resulting sparse linear systems is done using parallel computing on the parallel machines present in the ENEA Grid.

Several numerical experiments involving a simplified version of the NASA space shuttle (see Figure 1(b)) and incoming wave packets of the form:

$$(6) \quad u^i(\underline{x}, t) = e^{-[\underline{\gamma} \cdot \underline{x} - ct]^2 / 4\zeta^2}, \quad (\underline{x}, t) \in \mathbf{R}^3 \times \mathbf{R},$$

where $\underline{\gamma} \in \mathbf{R}^3$, $\|\underline{\gamma}\| = 1$ and $\zeta \neq 0$, have been performed (see [3]).

The original model of the NASA space shuttle shown in Figure 1(a) has been modified to obtain the simplified model of the NASA space shuttle shown in Figure 1(b). This simplification is necessary in order to represent using a single valued function the boundary of the obstacle in cylindrical coordinates. This last fact makes easy the implementation of the operator expansion method. The data relative to the original obstacle, Figure 1(a), are downloadable from the website <http://avalon.viewpoint.com/>. The reference surface $\partial\Omega_r$ and the internal surface $\partial\Omega_c$ have been obtained manipulating the 3D model shown in Figure 1(b).

We note that the physical dimensions of the shuttle are expressed in *units* where $1 \text{ unit} = 56.14/14 \text{ meters}$. The maximum length of the shuttle model shown in Figure 1(b) is of 14 units in the direction of the symmetry axis of its main body. This symmetry axis has been chosen as cylindrical axis (x_3 -axis) of the cylindrical coordinate system. Moreover we consider an acoustically hard space shuttle that is $\chi(\underline{x}) = +\infty$, $\underline{x} \in \partial\Omega$ and we recall that the sound speed in the air is $c = 331.45 \text{ meters/second}$ that corresponds to $c = 82.65 \text{ units/second}$. The ratio R_T of the boundary value

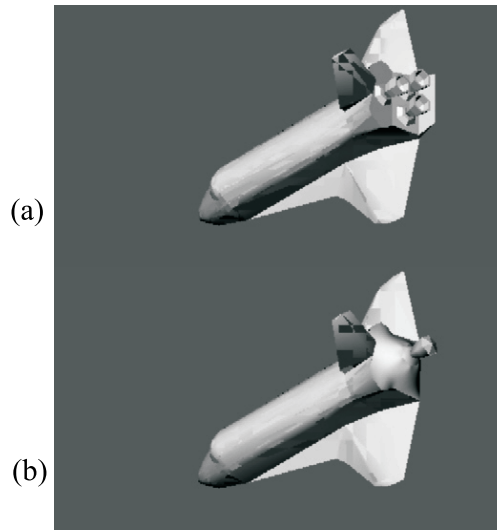


FIGURE 1. The NASA space shuttle (a) and its simplified version (b)

problems for the Helmholtz equation considered in these experiments ranges from a few tens to about one hundred.

Several animations and virtual reality applications concerning the numerical experiments discussed here and in [3] are shown in the website www.econ.univpm.it/recchioni/scattering/w12/. A more general reference to the work of some of the authors and of their coauthors in numerical scattering is the website www.econ.univpm.it/recchioni/scattering.

2. Cascading blackouts in a power transmission network

In the last years the frequent and sometimes impressive current breaks registered around the world made urgent the problem of understanding the behaviour of the high voltage power transmission networks in critical conditions in order to characterize adequate operations to protect their functions. Unfortunately the structural complexity of the real power transmission networks, due to the high dependence between the components of the network, complicates the study of their behaviour and is the main cause of their vulnerability. In fact, due to this interdependence, local overloads of lines or local failures of components can generate cascades of current breaks that, in extreme cases, can cause the fall of the whole network. We name this phenomenon: “cascading blackout”.

The blackout can be caused by inefficiency of the power transmission network or by excessive electric current demand from the loads that exceeds the network capacity. The cascading blackout is a sequence of events characterized by line failures and/or load sheds.

We represent a power transmission network as an undirected graph, that is as a set of nodes connected by branches. The nodes can be classified in three categories: generator nodes, representing wirings generating electricity, load nodes, constituting the load of the network, and junction

nodes. The branches connecting the nodes represent the network transmission lines where the electric power flows. The lines have their own characteristic admittance, small values of the admittance correspond to small power flow through the lines.

Let N be the number of nodes in the network considered and for $k, m \in \{1, 2, \dots, N\}$ let (k, m) be the line connecting the node k to the node m , in order to describe the electric power flow in the network the following quantities are considered:

- $y_{k,m} = g_{k,m} + ib_{k,m}$ the admittance associated to the line (k, m) , where i is the imaginary unit, $g_{k,m}$ and $b_{k,m}$ are, respectively, the conductance and the susceptance of the line (k, m) ,
- $\mathcal{P}_k^G$ the upper limit of the real power generated at the node k ,
- $P_k = P_k^G + P_k^L$ the active power injected into the node k , where P_k^G e P_k^L are, respectively, the real power generated and the real power demanded by the node k ,
- $F_{k,m}$ the power flowing through the line (k, m) ,
- $\mathcal{F}_{k,m}$ the upper power flow limit of the line (k, m) ,
- $P_D = \sum_{k=1}^N P_k^L$ the total load power demand,
- $P_C = \sum_{k=1}^N \mathcal{P}_k^G$ the total network capacity,
- $V_k = |V_k|(\cos\theta_k + \sin\theta_k)$ the complex voltage at node k .

Assuming assigned the network parameters mentioned above, in order to represent the power flow through the network, we solve the DC (Direct Current) optimal power flow problem, that consists in finding the vectors $\underline{P}^G = (P_1^G, P_2^G, \dots, P_N^G)$ and the corresponding voltage angles $\underline{\theta} = (\theta_1, \dots, \theta_N)$ that minimize the following cost function:

$$(7) \quad \Phi(P_1^G, P_2^G, \dots, P_N^G) = \sum_{k=1}^N P_k^G - W \sum_{k=1}^N P_k^L$$

subject to the constraints:

$$(8) \quad P_k^G - P_k^L = \sum_{m=1, m \neq k}^N F_{k,m}, \quad F_{k,m} = -b_{k,m}(\theta_k - \theta_m), \quad k, m = 1, 2, \dots, N,$$

$$(9) \quad |F_{k,m}| \leq \mathcal{F}_{k,m}, \quad 0 \leq P_k^G \leq \mathcal{P}_k^G, \quad \theta_1 = 0, \quad k, m = 1, 2, \dots, N,$$

where W is a real positive constant representing a cost associated to the load nodes (we choose $W=100$).

The optimal DC power flow model, describes the network behaviour in steady state regime when the values of the parameters characterizing the model (the load power vector $\underline{P}^L$, the generators and the lines capacities, the modulus of the generators voltages and the lines admittances) are assigned.

Changes in the parameter values vary the network behaviour eventually causing blackouts. We simulate the cascading blackout dynamics as a consequence of variations of the total load power demand P_D assuming assigned the values of the remaining parameters.

As the total load power demand P_D grows, the solution of the optimal DC power flow problem goes through some changes. The blackout happens when the DC optimal power flow problem (7)-(9) becomes unfeasible. In this case it is necessary to shed loads and/or to disconnect lines in the attempt of restoring feasibility of the model and in this way a new problem is formulated. If the new problem is also unfeasible, we continue to disconnect lines and/or to shed loads following some simple rules specified in [5] until we find a feasible problem and finally we solve the resulting feasible problem. This sequence of events generates the blackout cascade and mathematically corresponds to the formulation and the solution of a sequence of constrained optimization DC problems. That is we simulate with a numerical scheme the sequence of events that generates a cascading blackout.

We consider as blackout size measure the quantity: P_S/P_D where $P_S = \sum_{k \in \tilde{A}} P_k^L$, and $\tilde{A}$ denotes the subset of $\{1, 2, \dots, N\}$ that contains the indices corresponding to the loads shed as a consequence of the blackout cascade. We have $0 \leq P_S/P_D \leq 1$ (the maximum value 1 corresponds to total blackout).

Moreover in order to study the phase transition phenomenon in the network from a statistical point of view we analyze the network behaviour when the load power demand vector $\underline{P}^L$ is subject to random fluctuations.

Starting from a reference value of the load power vector $\underline{P}^L$ we overload the network adding to the components of $\underline{P}^L$ a random disturbance, that is we change the value of the total power demand P_D of a random quantity. As a consequence of that the resulting blackout size measure P_S/P_D becomes a random variable and we compute an approximation of its probability density function (pdf).

The analysis points out the existence of two values of the mean total power demand $P_D^{(1)}$, $P_D^{(2)}$, with $P_D^{(1)} < P_D^{(2)}$, of the mean total power demand P_D such that the blackout size measure probability density function corresponding to values $P_D < P_D^{(1)}$ is best fitted by a negative exponential function of P_S/P_D and the blackout size measure probability density function corresponding to values $P_D > P_D^{(2)}$ is best fitted by an inverse power function of P_S/P_D . This phenomenon is reminiscent of the phase transition phenomenon of equilibrium statistical mechanics. We note that the existence of the values $P_D^{(1)}$ and $P_D^{(2)}$ can be easily established numerically and that this is enough to confirm the existence of the phase transition phenomenon.

The particular case of the Italian high voltage power transmission network (Figure 2) has been studied and the simulations needed to approximate the blackout size pdf have been obtained with a FORTRAN code running on the ENEA Grid. These simulations are run using distributed computing, that is running clusters of hundreds of independent jobs on the grid.

The Italian high voltage power transmission network is represented as an undirected graph constituted of $N = 310$ nodes, with $N_G = 97$ generator nodes, $N_L = 163$ load nodes and $N_J = 30$ junction nodes, and $L = 361$ lines. Note that we will consider junction nodes as load nodes with power load smaller than $10^{-6} MW$. In this network there are 14 lines that are double lines (i.e. there are two lines joining the same two nodes). We substitute the double lines joining the corresponding nodes with a single line whose admittance is given by the sum of the admittances of the lines constituting the double line. Therefore the total number of lines in the network that we study is equal to

$M = 361 - 14 = 347$ (see Figure 2).

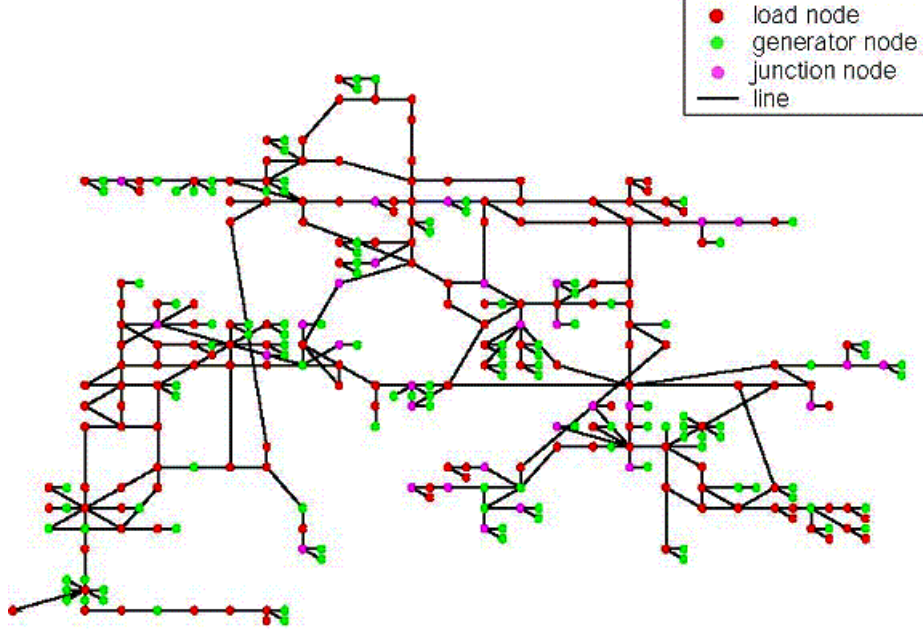


FIGURE 2. A graph representing an high voltage power transmission network

The numerical simulation of the network behaviour shows that the interval $[P_D^{(1)}, P_D^{(2)}] = [0.74P_C, 0.77P_C]$ can be chosen as critical interval.

Finally we study the line occupation. For $k, m = 1, 2, \dots, N$, we define occupation of the line (k, m) the value $|F_{k,m}|/\mathcal{F}_{km}$ when $\mathcal{F}_{k,m} > 0$, when $\mathcal{F}_{k,m} = 0$ the occupation of the line is not considered, in fact in this case the branch (k, m) of the graph does not represent a real transmission line of the network. When we consider a sample of power flows on a network we can compute the mean values of the line occupations of the sample. Figures 3 and 4 show for each line the expected value of the line occupation (on a sample of 20000 simulations) when P_D/P_C takes the critical values 0.74 and 0.77.

We can see that, in general, the line occupation is not homogeneous, that is the occupation of some lines is considerably greater than the occupation of the remaining lines of the network. This unbalance between line occupations can be the origin of large blackouts even when the mean value of the total power demand is smaller than the total network capacity. Note that the majority of the very loaded lines in Figures 3, 4, that is the lines whose occupation is close to one (the yellow lines in the figures), coincide. This fact suggests that these lines are the most vulnerable lines of the high voltage power transmission network considered in this study case.

The website <http://www.ceri.uniroma1.it/ceri/zirilli/w1> contains some auxiliary material including animations that helps the understanding of the phenomena studied.

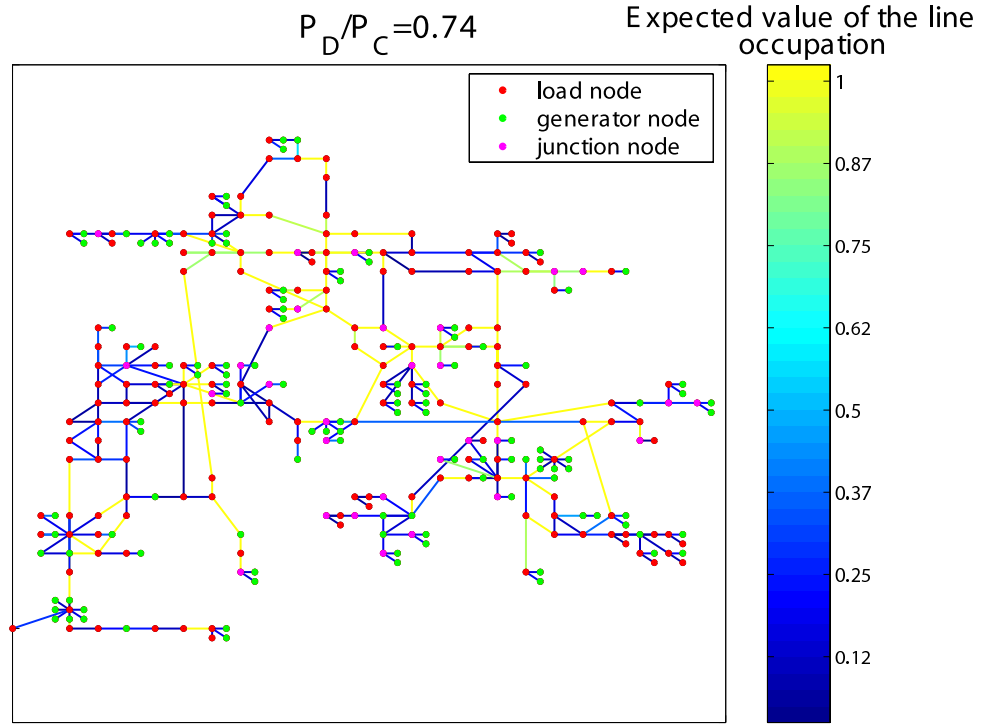


FIGURE 3. Expected value of the line occupations of an high voltage power network when $P_D/P_C = 0.74$

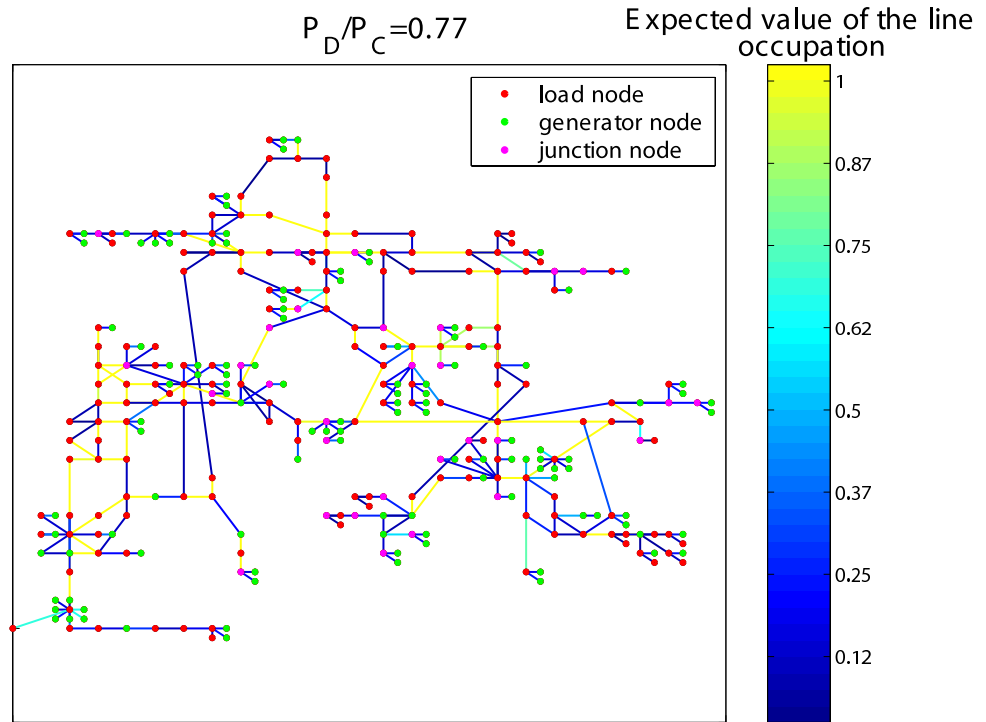


FIGURE 4. Expected value of the line occupations of an high voltage power network when $P_D/P_C = 0.77$

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